ECE 588

Fall 2018
Prof. George Gross
Room 4052 ECEB

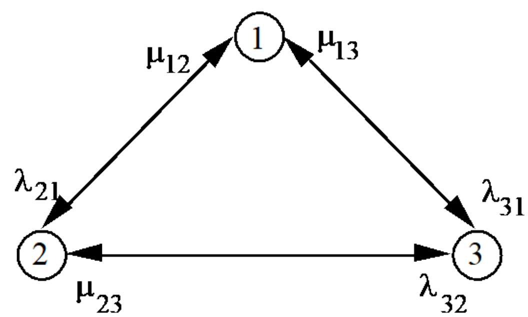
Homework 2

due Thursday, October 4, 2018

1. **[10 points] Prove** that

$$\mathcal{F}_i = p_i \sum_{j \neq i} \lambda_{ji}$$

2. Consider the following 3-state model

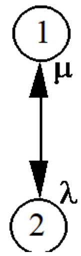


a) **[20 points]** Derive the expressions for:

$$p_1, \quad p_2, \quad p_3$$
$$\mathfrak{F}_1, \quad \mathfrak{F}_2, \quad \mathfrak{F}_3$$
$$\mathfrak{D}_1, \quad \mathfrak{D}_2, \quad \mathfrak{D}_3$$

in terms of the $\{\lambda_{ij}\}$, $\{\mu_{ij}\}$.

b) **[bonus 30 points – optional]** Suppose we are interested in representing the 3-state model by the simple 2-state model below



Under what conditions can we do such a reduced model representation? (Hint: we may conceptually consider state 3 as being absorbed into state 2; however, for a transition out of the augmented state 2, the transition intensity cannot be different for the former state 3 than the former unaugmented state 2.) **Derive** the expressions for $\mathscr{F}_i, \mathscr{D}_i$ and p_i, i = 1,2 under the permissible conditions.

3. Consider a system consisting of 5 *identical* generating units. Each unit has the following characteristics:

$$p = 0.96 \qquad c = 50\ MW$$
$$\lambda = 0.4/y \qquad \mu = 9.6/y$$

The generation unit failures occur (repairs are made) independently. In addition, an external event may occur that causes two generating units to fail at the same time. This type of failure, caused by an external event, is called a *common mode failure.* The external event occurs at the rate λ. Similarly, for the repair side, there are additional crews that enable two units to be returned to service at the same time at the rate μ.

a) **[10 points] Develop** the state transition representation under the condition that at any one point in time up to 2 units may be repaired or forced out. **Show** the model of the state space transitions.

b) **[20 points] Evaluate** all transition intensities in the model. **Determine** $\mathscr{F}_i, \mathscr{D}_i$ and p_i.

c) **[20 points] Evaluate** the cumulative distribution function

$$P\{\sum_{i=1}^{N} \underset{\sim}{A}_i \le x\}, \text{ where } N = 5 \text{ and}$$

$$\underset{\sim}{A}_i = \begin{cases} 0 & \textit{with probability } (1-p) \\ 50 & \textit{with probability } p \end{cases}$$

4. **[20 points]** For the simple Markov model of load with a cycle duration $\mathcal{D}_o$ and $\sum_{i=1}^{L} \alpha_i = 1$, **prove** that the following relations hold:

$$\lambda_{\ell_o +} = \frac{1}{(1-e)\mathcal{D}_o} \qquad \lambda_{\ell_i -} = \frac{1}{e\mathcal{D}_o} \qquad i = 1,2,\ldots,L$$

$$p_{\ell_o} = 1-e \qquad p_{\ell_i} = \alpha_i \, e$$

5. **[20 points]** Consider the example covered in class – the 5-level load model with the supply system consisting of the four identical units each 50 *MW* in capacity. **Verify** that the results presented are correct.

   Compute the expected unserved energy $\mathcal{U}$. You are requested to **state** the units in which $\mathcal{U}$ is expressed and **justify** the rationale for the units.

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